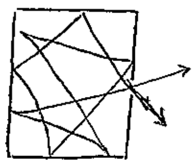
Failures of classical physics

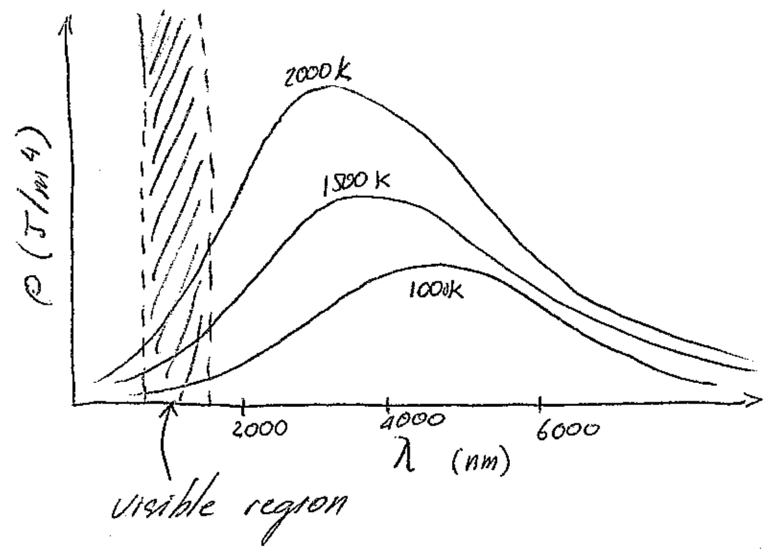
A. Black body radiation



A black body is an object with a low reflectivity and has capability of emitting and absorbing all frequencies of radiation uniformly.

Experiment:

Heat up a container with a pin hole then measure radiant emittance, ρ (energy per unit volume per unit wavelength) as a function of temperature and wavelength.



Observations:

a) The spectral radiant emittance has a maximum and the peak shifts toward shorter wavelength as the temperature is increased.

b) The total energy density, U (energy per unit volume), is proportional to the fourth power of the temperature

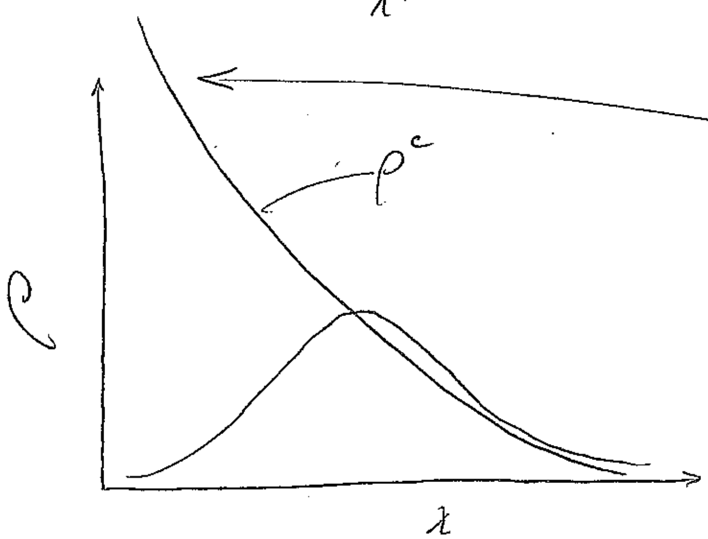
$$U = a\,T^4 \qquad \text{(Stefan's Law)}$$

a : constant.

Classical physics (Rayleigh-Jeans Law)

Assume that the blackbody radiation is a set of electromagnetic standing waves (behave like oscillators) inside the container. Each frequency of light is represented by one oscillator and the presence of light of a certain frequency ν is considered as the excitation of the oscillator of that frequency. Using classical equipartition theorem to calculate the average energy of the oscillators, Rayleigh-Jeans derived

$$\rho^c = \frac{8\pi kT}{\lambda^4} \qquad \rho \to \infty \quad \text{as} \quad \lambda \to 0$$



ultraviolet catastrophy (objects should glow in the dark !!!)

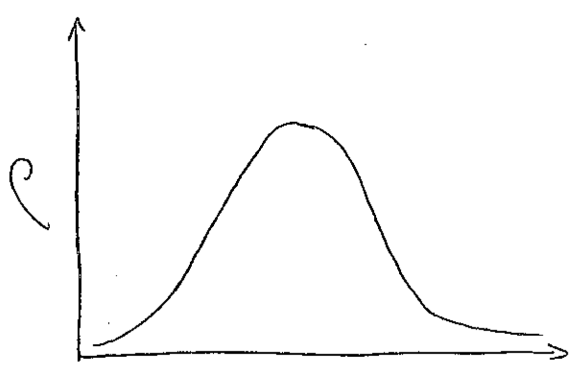
Plank's Distribution

Plank proposed "quantization of energy" that is each electromagnetic oscillator of frequency ν can access only discrete values of energy as multiple of $h\nu$, where h is a fundamental constant.

Light of frequency ν can be thought as consisting of 0, 1, 2 ... particles, called photons, each having an energy of $h\nu$.

With this assumption, Planck showed that

$$\rho = \frac{8\pi hc}{\lambda^5}\left\{\frac{1}{e^{hc/\lambda kT} - 1}\right\} \qquad h = 6.626 \times 10^{-34}\ \mathrm{J\ Hz^{-1}}$$



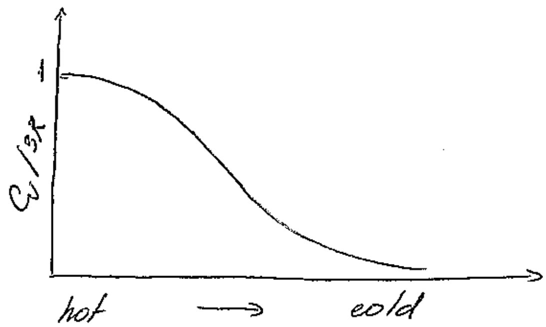
$\rho \to 0$ as $\lambda \to 0$

$\rho \to \frac{8\pi kT}{\lambda^4}$ as $\lambda \to \infty$

$\therefore$ $\lambda \to \infty$ $\Rightarrow$ classical limit

$\therefore$ light waves have particle-like properties!!

Heat capacity of Solids



The heat capacity of a solid is observed to decrease as the Temperature is decreased.

Atoms in solids have only vibrational motions.

Classical physics (use the equipartition theorem)

For 1-D harmonic oscillator : $H = \frac{P^2}{2m} + \frac{1}{2}kx^2$

The average energy of the oscillator is $2\left(\frac{1}{2}kT\right) = kT$

$\Rightarrow$ for 3-D oscillator, the average energy $= 3kT$

Thus, for N_A (Avogadro constant) atoms in a solid, the molecular internal energy

$$U_m = 3N_A kT = 3RT \quad (R: \text{gas constant})$$

$\Rightarrow$ The heat capacity

$$C_V = \left(\frac{\partial U_m}{\partial T}\right)_V = 3R = \text{constant}$$

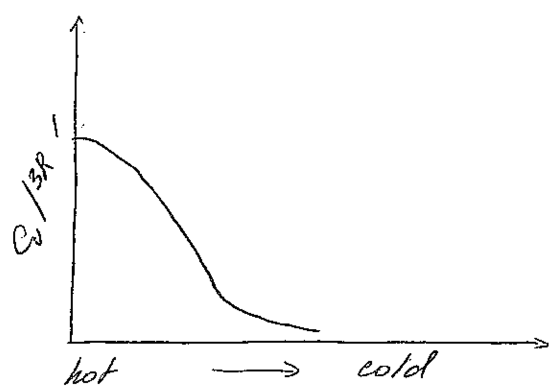
But C_V is observed $\to 0$ as $T \to 0$

Einstein formula

Assumed that each atom in the solid oscillates about its equilibrium position with a single frequency ν, then invoked Plank's energy quantization hypothesis that the energy of each oscillator is $nh\nu$.

$$U_m = \frac{3N_A h\nu}{e^{h\nu/kT} - 1} \qquad (10.1)$$

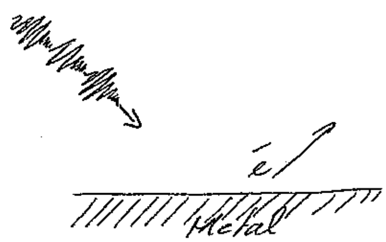
$$C_V = \left(\frac{\partial U_m}{\partial T}\right)_V = 3R\left(\frac{h\nu}{kT}\right)^2 \left\{ \frac{e^{h\nu/2kT}}{e^{h\nu/kT} - 1} \right\}^2 \qquad (10.2)$$



This form is qualitatively agreed with the experiment observation. Debye later allowed different atoms in the solid to vibrate with different frequencies and obtained better agreement.

Derive eq. (10.2) from eq. (10.1).

photoelectric Effect



Expose the metal surface to ultraviolet radiation then measure the kinetic energy of the ejected electrons.

classical physics expectation:

- More intense beam of light should eject electrons with greater energy.

Observations

a). Electrons are ejected only if the frequency of the light exceeds a threshold value characteristic of the metal, and regardless of its intensity.

b). The kinetic ~~of~~ energy of the ejected electrons is linearly proportional to the ~~incident~~ frequency of the incident light but independent of its intensity.

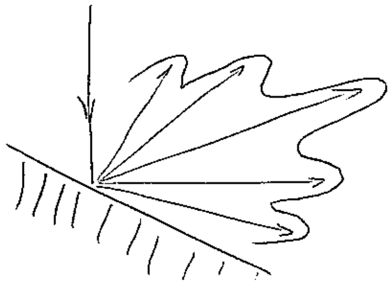
Based on Plank's hypothesis that light consists of discrete quanta, called photons, of energy $h\nu$, Einstein derived the expression for the kinetic energy of the ejected electrons

$$\frac{1}{2} m_e v^2 = h\nu - \phi \qquad (11.1)$$

where ϕ is the work function, the energy required to remove an electron from the metal.

$\therefore$ Light behaves like particles.

Electron diffraction

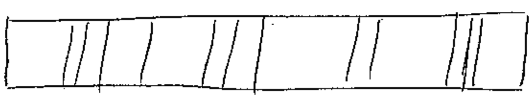


Davisson-Germer electron diffraction experiment showed particles (electrons) have wave-like properties with wavelength given by the de Broglie relation

$$\lambda = \frac{h}{p}$$

$\therefore$ Wave-particle duality of nature

Atomic and Molecular spectra



Atomic and molecular spectra showed that atoms and molecules absorbed and emitted radiation at discrete frequencies. This can only be understood if atoms and molecules are only allowed to access discrete energy levels. In classical mechanics, particles can access a continuous range of energy.

Heisenberg's uncertainty principle (1927)

It is impossible to specify simultaneously, with arbitrary precision, both the momentum and the position of a particle.

$$\boxed{\Delta x \, \Delta p \geq \hbar} \qquad h = 6.626 \times 10^{-34}\ \mathrm{J\,Hz^{-1}}, \quad \hbar = \frac{h}{2\pi}$$

Due to the small size of h, the uncertainty principle is only relevant for microscopic particles, like electrons.